ISEG - Lisbon School of Economics and Management

List of Exercises - Chapter 5
2^{nd} Semester of 2019/2020

March 27, 2021

1. If X and Y have the joint probability distribution

$$f(x,y) = 1/4, \text{ for all } (x,y) \in \{(-3,-5),(-1,-1),(1,1),(3,5)\}.$$

   Compute the $E(Z^2)$, where $Z = XY - 2X$.

2. Let X and Y be two random variables such that

$$f_{X,Y}(x,y) = \begin{cases} e^{-x-y}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$$

   Compute $E(XY)$ and $E(X)$.

3. If X and Y have the joint probability distribution

X/Y	-1	0	1
0	0	1/6	1/12
1	1/4	0	1/2

   Show that

   (a) $cov(X,Y) = 0$;

   (b) the two random variables are not independent.

4. If the probability density of X is given by

$$f(x) = \begin{cases} 1+x & \text{, for } -1 < x \leq 0 \\ 1-x & \text{, for } 0 < x < 1 \\ 0 & \text{, elsewhere} \end{cases}$$

   and $U = X$ and $V = X^2$, show that

(a) $cov(U, V) = 0$;

(b) U and V are dependent.

5. Let X_1,X_2, and X_3 be independent random variables with means 4, 9, and 3 and the variances 3, 7, and 5.

   (a) Find the means and the variances of $Y = 2X_1 - 3X_2 + 4X_3$ and $Z = X_1 + 2X_2 - X_3$.

   (b) Repeat (a) and (b) dropping the assumption of independence and using instead the information that $cov(X_1, X_2) = 1$, $cov(X_2, X_3) = -2$, and $cov(X_1, X_3) = -3$.

6. If the joint probability density of X and Y is given by

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{3}(y + x) & \text{, for } 0 < x \leq 1,\ 0 < y < 2 \\ 0 & \text{, elsewhere} \end{cases}$$

   Compute $Var(X|Y = y)$.

7. Let (X, Y) be a discrete random vector with joint probability function given by:

$$f(x, y) = \frac{x + y}{21}, \quad x = 1, 2; \; y = 1, 2, 3$$

   (a) Compute the means and variances of X and Y

   (b) Using $E(XY)$ analyze the independence of the two random variables and compute the correlation coefficient.

   (c) Compute $E(X|Y = 1)$

8. Suppose that $E(Y|X = x) = 9 + x$. If the $E(X) = 11$ and $Var(X) = 290$, what is $cov(Y, X)$?

9. Let X and Y be two continuous random variables such that the conditional density function of X given $Y = y$ is

$$f_{X|Y=y}(x) = \begin{cases} \frac{1}{2-y}, & y < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

   and the marginal density of Y is given by

$$f_Y(y) = \begin{cases} 1 - y/2, & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}.$$

   Compute the expected value of X.

10. Let (X, Y) be a two dimensional random variable, such that its set of discontinuities is $D_{X,Y} = \{(0,1), (0,2), (1,0), (1,1), (1,2)\}$ and its probability function is

$$f_{X,Y}(x,y) = \begin{cases} \frac{x+y}{a}, & (x,y) \in D_{X,Y} \\ 0, & \text{otherwise} \end{cases}.$$

(a) Find a and represent $f_{X,Y}$ by filling a suitable table.

(b) Compute $E(X)$ and $E(Y)$.

(c) Calculate $Cov(2X, 3Y)$ and $Var(X+Y)$.

(d) Characterize the distribution of $E(Y|X)$.

11. A company engaged in the trade of various items, whose sales have random behavior. The monthly sales of items A and B, expressed in monetary units, constitute a random vector (X, Y) with joint probability density function given by:

$$f_{X,Y}(x,y) = \frac{1}{2}, \quad 0 < x < 2,\ 0 < y < x.$$

(a) Compute the means and variaces of X and Y.

(b) Analyze the independence of the two random variables and compute the correlation coefficient.

(c) Find the $E(Y|X=1)$.

(d) Compute the mean and variance of total sales of the two articles.