1 Parametric Tests

1. Let X be a Normal random variable, $X \sim N(\mu, \sigma^2)$, with unknown mean and variance $^2 = 4$, and $(X1, X2)$ a random sample of size $n = 2$. In order to test $H_0 : \mu = 10$ against $H_1 : \mu = 14$ consider the following critical region $W = \{(x_1, x_2 : \bar{x} > 12.5)\}$ Compute the probabilities of type I and type II errors. Derive the critical region associated to the most powerful test of size 0.05.

2. Let θ be the proportion of voters that vote "yes" in a given referendum. With the purpos of testing $H_0 : \theta \leqslant 0.5$ against $H_1 : \theta > 0.5$, a random sample of 100 voters was drawn (all votes are expressed in "yes" or "no"). Let Y be the total of "yes" votes in the sample.

   a) Interpret the meaning of the proposed statistical test.

   b) Interpret, for this test, the meaning of type I and type II errors.

   c) Which of the following rejection regions is more appropriate (intuitively) as a decision rule for this test?

   - $W = \{y : y > 35\}$
   - $W = \{y : y < 45\}$
   - $W = \{y : y < 65\}$
   - $W = \{y : y < 25 \text{ or } y > 75\}$
   - $W = \{y : y > 55\}$

3. The life time of a certain type of electronic components, X, in hours, is assumed to have a normal distribution with standard deviation $\sigma = 50$. In order to test $H_0 : \mu = 250$ against $H_1 : \mu = 200$, the following rule was considered: reject H_0 if $\bar{x} < 230$.

   a) If the decision is based on a random sample of size 16, what are the significance level and the power of the test?

   b) What is the minimum sample size so that the probability of commiting type I error is smaller than 0.025?

4. A wine producer guarantees that the average acidity of their wine does not exceed 0.5 g/l. Suppose acidity follows a Normal distribution with unknown parameters.

   a) Based on a random sample of size n, propose a statistical test to check the guarantee of the producer.

   b) From a sample of 20 litres, an average of 0.7 g/l and a corrected standard error of 0.08 were obtained. What would you conclude about the guarantee?

5. An auditor assumes that the average amount of accounts receivable in a particular company is 750 euros. To support its statement he proposes to draw a sample of 36 accouts and to compute its average. He will only reject the assumed value of 750 euros if it is clearly contradicted by the sample mean, thus giving the benefit of the doubt to the value proposed.Assume that the amount of accounts receivable follows a normal distribution with standard deviation 125 euros.

   a) Define the rejection region for a significance level of 5%.

   b) Compute the p-value, assuming that a sample average of $\bar{x} = 800$ was observed.

   c) Assuming that a corrected variance of $s'^2 = 62000$ was observed, test whether the value given for the variance is plausible.

6. In order to assess the air quality in the two major portuguese cities two random variables were considering: X and Y, representing the number of airborne particles (micrograms per m^3) in Lisbon and Porto, respectively (the more the number of airborne particles, the worse the air quality). Suppose that the two random variables follow a normal distribution. Two samples were drawn: on of size 16 in Av. da Liberdade (Lisbon) and another of size 13 in Av. dos Aliados (Porto). The results were as follows: $\bar{x} = 92.9$, $s'_X = 25.4$, $\bar{y} = 86.1$, $s'_Y = 28.1$.

   a) Based on a proper statistical test, at the 5% level, show that we do not reject the equality of the variances of the two random variables X and Y.

b) Using the result of the previous question, compute the p-value of the statistical test to assess if the air quality is worse in Lisbon city center than in Porto city center.

7. A survey about the travel time to work for residents in the greater Lisbon area gave the following results for the Cascais and Barreiro areas:

	Cascais	**Barreiro**
Sample size	360	450
Average travel time to work (minutes)	35	44
Variance of the travel time	816	1202

Can we state, for a size of 0.05, that the average travel time is similar for those who reside in the two areas above?

8. A television chanel set an objective of having at least 55% viewers in prime time.

   a) From 100 viewers that inquired by telephone, during prime time, 51 answered they were viewing the television chanel. Propose an adequate test of hypothesis to test the television chanel objective and decide based on the p-value.

   b) The tv chanel used the services of a consulting market research firm to analyse if its objective is achieved. The market research firm will inquire, by telefone, 200 viewers every week, and the objective is considered not to be achieved if the number of positive answers is smaller or equal to 92. With this methodology what is the maximum value for the percentage of weeks in which the objective is incorrectly assumed not to be achieved.

9. Consider a random variable X with density function given by $f(x|\theta) = \theta e^{-\theta x}$ $(x > 0)$, for $\theta > 0$. A random sample of size 10 was selected in order to test $H_0 : \theta = 1$ against $H_1 : \theta = 1/2$. Two different decision rules were proposed:

Test 1 Reject H_0 when $\sum_{i=1}^{10} x_i > 15.705$

Test 2 Reject H0 when $\min(x_i) > 0.2996$

Compute the probabilities of committing type I and type II errors, and decide which rule is preferrable. Derive the critical region associated to the most powerful test of size 0.10.

10. Comment on the following statement: "If we reject the null $H_0 : \mu = \mu_0$ against a two-sided alternative, then we also reject H_0 against of a one-sided alternative, for the same test size."

11. In a given antional road 170 accidents were registered during the last 2 years (104 weeks). Assuming that the number of accidents per week follows a Poisson process with rate λ, is it plausible to state, at the 5% level, that the average number of accidents per week is at least 2?

12. The number of annual claims per policy can be modelled using a Poisson distribution with parameter λ. A random sample of size 1000 is collected to test $H_0 : \lambda = 0.1$ against $H_1 : > 0.1$.

   a) Derive the critical region associated to the uniformly most powerful test of size 0.05.

   b) Draw a graph of the power function associated to the above test for =0 .15 , 0.20 e 0.30.

13. The egg production of a farm can come from two categories, A and B. The eggs weight follows a Normal distribution with standard deviation $\sigma = 6$, and the mean weight for category A is 50 *grams*, while for category B is 55 *grams*. A supermarket orders a lot from category B, and once received the eggs would like to test whether the producer sent the right category. Consider $H_0 : \mu = 55$ against $H_1 : \mu = 50$.

   a) Having selected at random 12 eggs, whose total weight was 636 grams, should the supermarket complain? (test for both a level of 0.05 and 0.10).

   b) Compute the probability of committing a type II error, and interpret.

c) If you would like to have equal probabilities of committing errors of type I and II (0.05), what should the sample size be?

14. A car brand guarantees that a new model has an average city consumption that does not exceed 9.7 litres/100km. Suppose consumption follows a Normal distribution. From a random sample of size 10, a sample average of 9.88 and a corrected standard error of 1.6778 were obtained.

    a) Test the brand's guarantee for a level $\alpha = 0.01$.

    b) Compute the p-value for the above test using the given sample information.

    c) Supposing now that $\sigma = 1$, test the brand's guarantee for a level $\alpha = 0.01$.

15. tax revenue office has two cashiers. Suppose the time each cashier takes to attend a person follows a Normal distribution, both with standard deviation $\sigma = 2$ minutes. When Mr Brown arrives at the office, he finds 20 people queuing up at cashier A, while only 15 at cashier B, so he choses this second one. Mr Brown is called to be attended after 1 hour and 15 minutes, and at that time he notices that the 20th person at cashier A had just finished being attended. Can we state that the average attending time of the two cashiers is the same? (Test for both size 0.05 and 0.1).

16. A tourist office manager believes that during the summer season the current number of employees is not enough to attend customers, and more employees should be hired. The current number of employees is able to attend, on average, 50 customers per day; in the first two summer months (60 working days), an average of 51.8 customers per day was attended, with a corrected sample variance of 55.

    a) Use an appropriate test (size 0.05) to investigate whether it is necessary to hire more employees.

    b) What are the hypotheses underlying the previous question?

    c) Would your conclusions change if the number of customers attended per day followed a Poisson distribution?

17. The results of the football championships of two countries, say A and B, gave the following information about the total number of matches and the total number of scored goals:

Country	Total np. of matches	Total no. of goals
A	380	1178
B	306	866

    Supposing that the number of goals per match follows a Poisson distribution in both countries, test the hypothesis that the average number of goals per match is the same in the two countries.

18. In order to study wage discrimination between men and women working their first year in consulting firms, a researcher collected two independent samples, having obtained the following results:

Sample	Average annual wage	Standard error	Sample size
Women	43 217 euros	12 560	125
Men	47 121 euros	17 654	100

    Test, for $\alpha = 0.05$, the hypothesis that there is no wage discrimination.

19. The expected deadline for receiving payment of the invoices in a certain firms is equal to 60 days. Having selected at random 45 invoices, the firm observed an average payment time of 85 days, with a corrected sample variance equal to 6700. Test whether the expected deadline was exceeded, using a size of 0.10. What is the power of the previous test if the actual average deadline were of 75 days?

20. The Revenue and Customs would like to assess whether the tax return forms filed online present less errors that those sent by paper. In two random samples of 500 online forms and 450 paper forms, the number of forms with errors were, respectively, 125 and 128. Perform an appropriate test and interpret the result.

21. The effect of an energy-saving campaign was assessed selecting a random sample of 25 families, whose energy consumption before and after the campaign was observed, say X_1 and X_2 respectively. For each family, the consumption difference $D = X_1 X_2$ was computed, having obtained an average of 0.2 kWh and a corrected standard error $s'_D = 1$ kWh. Suppose monthly consumption follows a Normal distribution.

   a) For a size of 0.05, test whether the campaign was effective.

   b) If the sample values reported above were coming from a sample of size 225, would your conclusions be the same?