1 Sampling and Sampling Distributions

1. In a particular university 30% of students have car. A random sample of 20 students is selected from the population.

   a) What is the probability that the first student selected has a car and the others do not?

   b) What is the probability that no more than 10 students have a car?

   c) Compute the expected value and the variance of the proportion of students, in the sample, that have a car.

2. The number of misprints per page, in a certain type of publications, is a random variable with Poisson distribution whose average is estimated to be 0.3. It is assumed that there is independence between the number of misprints on different pages.

   a) What is the probability that, in a sample of 5 pages, that each of the first two have a misprint and the others do not have misprints?

   b) For a random sample of 20 pages, calculate the probability that the total number of misprints found is at least 8.

   c) For a sample of 50 pages, compute the expected value and variance of the average number of misprints in the sample.

   d) For a sample of 5 pages, compute and interpret $P(\max(X_1) \leqslant 1)$

   e) In a publication with 100 pages, what is the probability that at least 80 of them have no misprints?

3. Consider a population X with probability function given by

$$f_X(x) = \frac{1}{3}, \qquad x = 0, 1, 2$$

   A random sample of size 3 is selected. Find the distribution of the maximum and the minimum of the sample.

4. Consider a population X with probability function given by

$$f_X(x) = 3x^2, \qquad 0 < x < 1$$

   from which a random sample of size 5 is collected. Determine the probability that the maximum value in the sample is lower than 0.9

5. Consider the previous exercise (exercise 4) and define the variable N representing the number of elements in the sample not exceeding 0.9.

   a) Determine the distribution of N.

   b) Answer question 4 using variable N.

6. There are five daily flights departing from A and arriving at B. The delay in the departure time, in minutes, is a random variable following an exponential distribution with mean 15 minutes. How likely is it that the lowest delay, in a day, is less than 18 minutes? How likely is it that the longest delay , in a day, is higher than 30 minutes?

7. Fifteen amplifiers, that were simultaneously connected, were selected randomly. Assuming that the lifetime of one amplifier follows a normal distribution with mean 1000 hours and standard deviation 200 hours, determine the probability that the first amplifier to crash has a life span exceeding 800 hours.

8. Let $(X_1, \ldots, X_n)$ be a random sample from a population that follows an exponential distribution with parameter θ.

   a) Find the distribution of the sample mean.

b) Compute $P\left(\bar{X} > \dfrac{2}{\theta}\right)$

c) Find an expression for $P(X_1 \leqslant c, X_2 \leqslant c, \ldots, X_n \leqslant c)$ and relate the result with the distribution of the maximum and the minimum of the sample.

9. The time, in minutes, that an employee in the canteen takes to serve a person is a random variable with probability density function given by

$$f_X(x) = e^{-x}, \qquad x > 0$$

If at 14:00 forty people are on the waiting line of the canteen, what is the probability that none is served at 14:30.

10. Consider that the run time of a piece is a random variable with exponential distribution with mean 5 minutes.

a) Write the expression of the probability density function of a random sample of size 5.

b) For that sample, calculate the probability that the run time of the first two pieces of the sample is less than $8\,min$ and the run time of the others is higher than $8\,min$.

c) If 5 pieces are taken at random, calculate the probability that two of them have a maximum run time of $8\,min$. Compare with the result in b).

d) For a sample of size 100, obtain the distribution of the sum and of the average of the sample. Calculate their means and variances.

e) Comment on the following statement: "in 90% of the samples of size 100, the mean run time is less than $10\,min$."

11. A sample of four lamps, that were connected together, was selected. It is known that the life span of one lamp follows a normal distribution with mean 800 hours and standard deviation 100 hours.

a) Determine the probability that the first lamp to crash has lasted more than 900 hours.

b) How likely is it that the last lamp to crash has been burning for more than 1000 hours?

c) What is the probability that the average life span of the sample lamps differs from the mean life span of the population by more than 100 hours?

12. The time a student spends per day on messenger is assumed to follow an exponential distribution with mean 2 hours. Five days were selected at random and the time spent on messenger was observed in each of the 5 days.

a) Calculate the probability that, for the 5 days observed, the average time spent on messenger per day is longer than 4 hours.

b) How likely is it that the maximum observed time spent on messenger per day is less than 6 hours?

13. Let X be a random variable with probability density function given by

$$f_X(x) = 2x, \qquad 0 < x < 1$$

and let $(X_1, \ldots, X_{36})$ be a random sample of this populations.

a) Compute $P\left(0.4 < \bar{X} < 0.6\right)$.

b) Find k such that $P(\bar{X} \leqslant k) = 0.95$.

14. From past experience it was found that 5% of IRS delivered statements contained illegal deductions. For control purposes, 1000 claims were randomly examined. Assuming that the pattern of previous years still holds, calculate the probability that at least 60 have such illegality.

15. Assume that the probability that a randomly chosen student has an opinion favorable to the existence of lectures is 0.5.

a) In a random sample of 50 students, what is the approximate probability of observing more than 35 students with a favorable opinion?

b) What is the minimum number of students that must be surveyed so that the difference between the relative frequency of favorable students in the sample and the actual ratio of favorable students is less than 2% with probability of at least 95%?

16. A distributor of whiskey knows from experience that the probability of counterfeit bottles on the market is 0.05. 1000 bottles are checked regularly, at points of sale. With at least 98% probability, find the maximum admissable deviation between the relative frequency of the faked bottles and the true proportion of faked bottles?

17. Assuming that 1 in 5 students wear glasses, what is the probability of observing more than 30% of students with glasses in a sample of size 20? And in a sample of size 50?

18. In a sports club, the proportion of adherents with a favorable opinion on the direction is 75%.

    a) In 1000 fans randomly selected, what is, approximately, the probability of observing less than 720 with a favorable opinion on the direction?

    b) What should be the minimum size of a sample so that the deviation between the sample relative frequency and the true proportion does not reach 0.02 in 95% of cases?

19. When starting a campaign for a referendum, the "yes" has, in the population, a ratio of 53% of voters. Following the end of the campaign this proportion decreased to 49%. If a sample of size 100 was collected at the beginning of the campaign and another sample of size 120 was collected at the end of the campaign, how likely is it that the sample values indicate a growth of "yes" voters.

20. After a vigorous advertising campaign, the market share of a certain brand of potato chips increased from 8% to 10%. Suppose that two surveys were launched, one before the start of the campaign (sample size of 100) and another two weeks after the end of the campaign (sample size of 300).

    a) How likely it is to conclude, based on these surveys, that the gain in market share was bigger than 5 percentage points?

    b) How likely is it that the surveys lead to the conclusion of a loss on the market share.

21. Consider two Bernoulli populations with parameters θ_1 and θ_2. If a sample of 50 observations is taken from population 1 and a sample of 40 observations is taken from population 2, what is, approximately, the probability that the difference between the two sample proportions is, in absolute value, greater than 0.2?

22. A random sample of size 5 is selected from a normal population. Compute the probability that the sample standard deviation is lower than the population standard deviation.

23. A random sample of size 100 was selected from a normal population with mean 8 and standard deviation 4.

    a) Compute the probability that the sample mean differs from the population mean by more than 0.5.

    b) If the population was not normal, what would this probability be?

24. Let $X \sim N(20, 25)$. Compute the size of the sample so that the probability that the sample mean is between 18 and 22 is at least equal to 0.9.

25. The person responsible for the quality control in a certain factory established that a machine is out of tune if, in a random sample of 16 pieces selected from the daily production, the average length observed is greater than 61.1 or less than 58.9. Assume that the length of the pieces, when the machine is tuned, follows a normal distribution with mean 60 and standard deviation 3.

    a) What is the probability that the machine is wrongly regarded out of tune?

    b) What should be the minimum sample size so that the probability that the machine is considered unduly out of tune is less than 5%?

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26. Let T be a random variable following a $t - Student$ distribution with 10 degrees of freedom.
    - **a)** Find a such that $P(T < a) = 0.05$
    - **b)** Compute $P(T < 0.26)$
    - **c)** Find b such that $P(T > b) = 0.9$
    - **d)** Find c such that $P(-c < T < c) = 0.95$

27. Let X be a random variable following a $F - Snedcor$ distribution with 8 and 20 degrees of freedom.
    - **a)** Find a such that $P(T < a) = 0.95$
    - **b)** Find b such that $P(T < b) = 0.05$
    - **c)** Compute $P(0.25 < X < 2.45)$

28. Let X and Y be independent random variables such that $X \sim N(0, 3)$ and $Y \sim \chi^2_{(60)}$.
    - **a)** Find a such that $P(X < a\sqrt{Y}) = 0.95$
    - **b)** Compute $P(5X^2 < Y)$

29. Let X, Y and W be independent random variables such that $X \sim N(0, 2)$ and $Y \sim \chi^2_{(10)}$ and $W \sim \chi^2_{(5)}$.
    - **a)** Compute $P\left(\dfrac{X}{\sqrt{Y}} \leqslant 1\right)$
    - **b)** Find b such that $P(Y > bW) = 0.95$

30. A random sample of size 4 was selected from a normal population with unknown mean and variance.
    - **a)** Determine the percentage of samples in which the sample mean differs from the population mean by values greater than the standard deviation of the population.
    - **b)** Determine the percentage of samples in which the sample mean differs from the population mean by values greater than the sample standard deviation.

31. A researcher intends to estimate the mean of a normal population using the sample mean. What is the probability that the difference, in absolute value, between the sample mean and the mean of the population is less than half of the corrected sample standard deviation? What should be the minimum sample size so that such probability is higher than 90%?

32. A sample of size 3 was selected from a normal population with variance equal to 64. How likely is it that the sample variance is greater than 78? (Consider a sample size of 16.)

33. A center security card issuer has two personalization equipments operating independently. The processing time, in seconds, for each of them follows a normal distribution with the same mean and standard deviation equal to 10 and 15 seconds, respectively. Taking samples of size 16 from both equipments, compute the probability that the difference between the sample means, in absolute value, is greater than 5.

34. The fuel consumption, in liters per 100 km, of cars from brands A and B are independent random variables X and Y, respectively, that follow normal distributions with $\mu_X = 8.1$, $\sigma_X = 1.2$, $\mu_Y = 7.7$ and $\sigma_Y = 1.5$. A potential buyer will observe the consumption of a random sample of 25 cars from each brand and decide to buy the one with lower average consumption. What is the probability that the buyer buys a car from brand A?

35. Two samples of size 100 were taken from two normal populations with equal means and variances $\sigma_1^2 = 50$ and $\sigma_2^2 = 40$. Compute the probability that the difference, in absolute value, between the sample means is greater than 2.

36. Two samples of size 10 and 5, respectively, were collected from two normal populations with equal variances. Find values a and b such that the probability that the ratio of the corrected sample variances is between those values is 95%.

37. Let X and Y represent two normally distributed populations such that $\mu_X = 20$, $\mu_Y = 22$, $\sigma_X^2 = \sigma_Y^2 = 16$. Independent random samples of size 9 and 16, respectively, were selected.

    a) Find the probability that the mean of the sample from population Y exceeds the mean of the sample from population X by more than three.

    b) Compute the probability that the sample corrected standard deviation from population X is bigger than the double of the sample corrected standard deviation from population Y.